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| **ECE 3300 Exam 5 Notes Sheet — spring 2017 — Revision 6** | | | | | |
| Laplace Transform Properties w/ ROC | | | Laplace/Z Transform of Real Signals  If is real valued, does not depend on except through .  If is real valued, does not depend on except through . | |  |  |  | | --- | --- | --- | |  | EVEN |  | |  | ODD |  | |  | EVEN |  | |  | ODD |  |   If is purely even or odd, ROC has the form where  If is purely even or odd, ROC has the form where | Inverse Laplace Transform Steps   1. Partial fraction expansion. 2. Determine all ROCs 3. For each ROC, determine the corresponding   Inverse Z Transform Steps   1. Partial fraction expansion using substitution 2. Determine all ROCs 3. For each ROC, determine the corresponding |
| **Laplace/Z Transforms Involving Unit Step** | | | (**Lightest**): ROC to the right of the rightmost pole: right-sided.  (**Medium**):Between 2 poles: is 2-sided  (**Darkest**): ROC to the left of leftmost pole: left-sided | Factors Raised to an Integer Power  If , add the following terms to the partial fraction expansion: | - |
| Z Transform Properties w/ ROC | | | (**Lightest**): ROC outside outermost pole: right-sided  (**Medium**):ROC between 2 poles: is 2-sided  (**Darkest**): ROC inside innermost pole: left-sided. | Improper Fractions | **Fundamental Laplace/ Z Transforms** |
| Laplace/Z Transforms with Quadratic Factors |  | | | Inverse Transforms Involving Quadratic Factors |  |
|  | Laplace/Z/Fourier Transform of Periodic Signals | | | *Further explanation below* |  |
| PARTIAL FRACTOIN EXPANSION EXAMPLE |  | Transform Note  Becomes… | | This transform was in a question on a previous exam (F2015), so it could be useful. | **Important Partial Fraction Expansion Rule**  If the numerator has degree *n* and the denominator has degree *m*, there are additional terms from and so on up to . This is also true with Laplace transforms; in this case the terms are and so on up to .  Examples:  Does not happen here! |
| **Classifying Signals: Z Transform**  The way to approach problems like this is through the ROC of the Z transform. If necessary, break the signal into two pieces, one for and the other for . If the ROC **includes** , then **both transforms exist and** . If the ROC **borders but does not include (it can border)**, then **both transforms exist but** . If the ROC **does not even border** , then the **Z transform exists but the Fourier transform does not**. If there is no ROC, then no Z transform exists; in this case, the Fourier transform exists if the Fourier transforms of the two sides exist. (Periodic signals are one way this happens.) | | | | **Classifying Signals: Laplace Transform**  If necessary, break the signal into two pieces, one for and the other for . If the ROC **includes** , then **both transforms exist and** . If the ROC **borders but does not include (it can border)**, then **both transforms exist but** . If the ROC **does not even border** , then the **Laplace transform exists but the Fourier transform does not**. If there is no ROC, then no Laplace transform exists; in this case, the Fourier transform exists if the Fourier transforms of the two sides exist. (Periodic signals are one way this happens.) | |

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**Examples**

(F15, 3) Suppose the signal has Z transform . Suppose with the same ROC as that of . Express as a function of .

(F15, 5) Suppose . If the ROC is , determine . What is ?

(F15,8) Find if with ROC

(F15,9) Suppose has Laplace transform X(s) with ROC 1 < Re{s} < 9 and. Determine the ROC for Y(s).

(S16,4) Suppose . Find C in the partial fraction expansion. PFE has the form

(S16,6) Suppose has Z transform with ROC , , and Determine ROC for Z transform of .

(S16,8) Suppose the signal has the Z transform X(z). Further suspose . Determine . What is ?